Part I: Lévy processes and their product systems

Volkmar Liebscher

This lecture will introduce the essential theory of stochastic processes to provide necessary background for other lectures, in particular Tsirelson's lecture in the second week. Naturally, Lévy processes as processes with stationary and independent increments are closely related to notions of independence. On the level of the L^2-spaces, independence directly gives a tensor product structure basic to product systems. General (Lie-)group valued Lévy processes form a bridge to quantum Lévy processes. Following work of Tsirelson, semigroup-valued Lévy processes are a valuable example for constructing type II product systems. The connection to Markov processes as function valued Lévy processes is also explored.

I. $\mathbb{R}$-Valued Lévy processes

1. Brownian motion
2. Poisson processes
3. Fock spaces as their product systems
4. Lévy processes: Definition and representation via Brownian motion and compound Poisson process

II. Group-valued Lévy processes

1. Definition, examples
2. Algebraic representation
3. General construction of the product system
4. Type of the product system

III. Semigroup-valued Lévy processes

1. Definition
2. Realisation of Markov processes
3. Construction of the product system
4. Examples of type II product systems

Part II: Type II product systems

Volkmar Liebscher

Type II product systems of Hilbert spaces are those which possess at least one unit, but whose units do not generate the whole space. This lecture will show that the units provide close links between so-called stationary factorising measure types of random sets and invariants of product systems. Also, it will become clear that there is a universe of product systems beyond the type I product systems obtained from classical group-valued Lévy processes. As application, the structure of the spatial product of product systems is explored.