International Conference

"Product Systems and Independence in Quantum Dynamics"

March 4-18, 2011, Alfried Krupp Wissenschaftskolleg Greifswald

Abstracts

O. Arizmendi

Commuting Relations between Boolean and Free Convolutions

Belinschi and Nica introduced a composition semigroup defined on the set of probability measures. Using this semigroup, they also introduced the free divisibility indicator, from which one can judge if a probability measure is infinitely divisible or not. To prove the properties of this composition semigroup some commuting relation between boolean and free additive convolution is fundamental. In this talk we prove that there are other commutation relations involving free and Boolean powers. Using this we introduce a composition semigroup for multiplicative convolutions which behaves similar to the additive case.. We also study how the indicator changes with respect to free and Boolean powers (for both multiplicative and additive case) . As a byproduct, we prove Bozejkos conjecture: the tth Boolean power of a probability measure preserves free infnite divisibility for $0 \leq t \leq 1$. A similar result is obtained for the multiplicative case.

B.V.R. Bhat

Endomorphism Semigroups from Dilations

M. Bozejko

q-Gaussian Random Variables with Application

We will present the following subjects:

1. q-Gaussian and theta function of Jacobi- formula of Szablowski.
2. Free infinite divisibility of q-Gauss $(0 < q < 1)$, Normal law $(q = 1)$ and Askey-Wimp-Kerov distribution.
3. q-Gaussian for $q > 1$ and applications to combinatorics.

F. Fidaleo

Ergodic Properties of Bogoliubov Automorphisms in Free Probability

We show that the Bogoliubov automorphisms on the $C*$-algebra generated by the (self-adjoint part of the) annihilators associated to the Free Commutation Relations enjoy the strong ergodic properties of unique ergodicity, unique weak mixing, or unique mixing. In addition, it is possible to construct models enjoying such strong ergodic properties whose GNS representation of the unique invariant state (the Fock vacuum) generates all the von Neumann factors except the type II_∞ and III_0. The above results are extended to the q-Commutation Relations.

R. Floricel

Boundaries of CP-Semigroups and E-Semigroups

U. Franz

Positive Definite Functions on Algebraic Quantum Groups

M. Gäbler

Fock Space, Factorisation and Beam Splittings - With Application to a Quantum Model of Brain Activity

Neuroscientific phenomenona like the so-called binding problem (synchronised action of neurons) and the incompatibility of EEG and MEG measurements suggest that brain activity might be governed by quantum principles. We present ten postulates derived from neuroscience any such quantum model of recognition of signals should comply with. Two of these postulates are of a particular interest, namely the decomposition of signals and their parallel processing according to different brain areas, which lead to a factorisation property of both the space of signals and the processing operator. We show that general beam splittings on multiple symmetric Fock space are therefore a good choice for this modelling.

M. Gerhold

Additive Deformations and Quantum Lévy Processes of Braided Hopf Algebras

Additive deformations of bialgebras are motivated by the Heisenberg algebra, i.e. the algebra with generators a, a* fulfilling the relation $aa* - a * a = t1$. Defining a comultiplication, which is primitive on the generators gives a bialgebra like structure, but it now is a homomorphism from this algebra to the tensor product for different values of t.

An additive deformation is a real indexed family of multiplications on a bialgebra which is in a certain sense compatible with the comultiplication. This allows to convolve states with respect to the different multiplications and one can generalize the Schoenberg correspondence, which states a 1-1-correspondence between conditionally positive functionals and semigroups of states.

F. Hiai

Generalizing Minkowski's Determinantal Inequality

The famous Minkowski's determinantal inequality is

$$\det^{1/n}(A+B) \geq \det^{1/n}A + \det^{1/n}B$$

for $n \times n$ positive semidefinite matrices A, B. In this talk we discuss some types of generalizations of Minkowski's inequality. First, we generalize Minkowski's inequality to the forms with a convex or concave function. If $g : [0,\infty) \to [0,\infty)$ is convex with $g(0) = 0$ and A, B are $n \times n$ positive semidefinite matrices, then

$$(*) \quad \det^{1/n}g(A+B) \geq \det^{1/n}g(A) + \det^{1/n}g(B).$$

If f is a non-negative concave function on an interval Ω and A, B are $n \times n$ Hermitian matrices with spectra in Ω, then

$$(**) \quad \det^{1/n}f\left(\frac{A+B}{2}\right) \geq \frac{\det^{1/n}f(A) + \det^{1/n}f(B)}{2}.$$

A class of functionals on matrices, called symmetric anti-norms, is introduced, including the Schatten r-norms for $r \in (-\infty, 1] \setminus \{0\}$ and $\Delta_k(A) := \left(\prod_{j=1}^{k} \mu_{n+1-j}(A)\right)^{1/k}$, $k = 1, \dots, n$, for $n \times n$ matrices A, where $\mu_1(A) \geq \cdots \geq \mu_n(A)$ are the singular values of A. Let $g, f : [0,\infty) \to [0,\infty)$, $0 < q < 1 < p$, and let A, B be positive semidefinite matrices. Then the subadditivity/superadditivity inequalities

$$\|g(A+B)\|^q \leq \|g(A)\|^q + \|g(B)\|^q,$$
$$\|g(A+B)\|_!^p \leq \|g(A)\|_!^p + \|g(B)\|_!^p$$

are shown when $\|\cdot\|$ is a symmetric (unitarily invariant) norm and $\|\cdot\|_!$ is a symmetric anti-norm.

Let τ be the normalized trace on matrices. Let φ be a continuous function on an interval Λ. We characterize the function φ ensuring that if $f : \Omega \to \Lambda$ is a function on an interval Ω such that $\varphi \circ f$ is convex (resp., concave) then $A \mapsto \varphi \circ \tau \circ f(A)$ is convex (resp., concave) on Hermitian matrices with spectra in Ω.

Finally, we introduce functionals $\Delta_t(x)$, $t \in (0, \infty)$, extending the Fuglede-Kadison determinant, for τ-measurable operators x affiliated with a semifinite von Neumann algebra $\mathcal{M}$ with a faithful normal semifinite trace τ. We extend the above inequalities (*) and (**) to the functionals Δ_t.

R.L. Hudson

Quantum Lévy Area

We replace the two independent Brownian motions in the classical definition of Lévy area by the mutually noncommuting momentum and position Brownian motions of quantum stochastic calculus. For the quantum Lévy area formula foir the characteristic function, the correct analog is obtained by replacing the exponential of the double integral by the causal double product integral generated by the infinitesimal of area (which is the same thing classically). This double product is found explicitly as the second quantization of a limit of a discrete double product of rotation matrices in the Fock case and by extension using Shale's theorem also in non-Fock quantum stochastic calculus. For the latter the quantum Lévy area formula is nontrivial. It is conjectured that it is a Meixner charateristic function.

M. Junge

Markov Dilation with Continuous Path

We will show that selfadjoint semigroups of completely positive unital and trace preserving maps admits a Markov dilation. This result extends former results of Kümerrer and Maassen for semigroups on matrix algebras. We will use tools from free probability in the general case. It turns out that existence of a Markov dilation with continuous path is equivalent to the existence of a derivation with sufficiently many smooth elements (in the sense of Sauvegeot). If time permits we discuss some applications to rigidity.

This is joint work with Ricard and Shlyakhtenko.

A. Kula

Lévy Processes on Compact Quantum Groups and Their Symmetries

V. Liebscher

The Product Systems of Bessel Zeros

Based on the notion of a stationary factorisable measure class of random closed sets we analyse the product systems coming from the zero sets of Bessel processes. Especially, we analyse their spatial and tensor product completely and show how they differ.

J.M. Lindsay

Quantum Stochastic Integrals and Semimartingales

A minimal quantum dynamical semigroup on the full algebra of all operators on a Hilbert space $\mathfrak{h}$, in the sense of Davies (after Kato and Feller), is specified by its infinitesimal data — a pair of operators consisting of the generator K of a contractive C_0-semigroup on $\mathfrak{h}$ and an operator L from $\mathfrak{h}$ into $\mathfrak{h} \otimes \mathsf{k}$, for another Hilbert space k, which together satisfy a dissipativity relation.

In this talk I show how, under a sectorial condition on K, the quantum dynamical semigroup may be 'dilated' by means of a quantum stochastic contraction cocycle on $\mathfrak{h}$ driven by quantum noise with multiplicity space k. Necessary and sufficient conditions will be given for the dilation to be an E-semigroup (respectively, an E_0-semigroup), in terms of (two families of) $(d+1)$ associated semigroups where $d = \dim \mathsf{k}$.

D. Markiewicz

E_0-Semigroups Arising from Boundary Weight Maps of Finite Range Rank

In the CP-flows approach introduced by Powers, E_0-semigroups are obtained from certain maps called boundary weight maps. I will present some results obtained in joint work with Jankowski, regarding boundary weight maps of a specific form giving rise to type II_0 examples, including the explicit description of their gauge groups. I will then describe a generalization of those results obtained in joint work with Jankowski and Powers, to the case of boundary weight maps which have finite dimensional range in the appropriate sense.

M. Mokhtar-Karroubi

Compactness and Spectral Theory of a Class of Quantum Dynamical Semigroups

This work deals with an abstract class of quantum dynamical semigroups whose generators have general (unbounded) hamiltonian part and bounded interacting part. We provide a systematic analysis of compactness problems pertaining to spectral stability problems; in particular, we analyze the stability of essential spectra or essential types for quantum dynamical semigroups in the Banach space $\mathcal{L}^1$ of self-adjoint trace class operators. We also study the exponential trend to equilibrium state for some trace preserving quantum dynamical semigroups. Finally, much more precise compactness results are given in the space of self-adjoint operators in the Schatten class $\mathcal{L}^p$ $(1 < p < \infty)$; in particular, non-commutative "averaging lemmas" are given.

M. Mukherjee

Isomorphism Class of Amalgamated Product Through Contractive Units

M. Perrin

H_p-Theory for Continuous Filtrations

E. Ricard

On Markov Dilations

O.M. Shalit

Subproduct Systems

Subproduct systems arose in the study of quantum dynamical semigroups. In this talk I will explain two ways in which they arise, and I will overview some applications to dilation theory, as well as connections to operator algebras.

This talk is based on joint works with B. Solel, M. Skeide, and K. Davidson and C. Ramsey.

M. Skeide

Dilations of Markov Semigroups with the Help of Product Systems

We explain how the GNS-product system of a Markov semigroup can be used to dilate that Markov semigroup. If (and only if) the Markov semigroup is spatial, the dilation is a cocycle perturbation of a noise.

We explain that every noise is a monotone noise, and that every monotone noise can be "blown up" to a free noise.

B. Solel

Non Commutative Function Theory

Sometimes an operator algebra can be viewed usefully as an algebra of operator-valued functions on its space of representations. I claim that for the class of tensor algebras and their ultraweak closures this can be done in a way that justifies the view that the theory of these algebras is indeed "Non commutative function theory". The purpose of the talk is to try to present enough evidence to convince you that this is indeed the case. I will introduce these algebras, describe their representations (and how to parametrize the representation spaces in a useful way) and determine what are the functions that the elements of the algebra give rise to.

This is a joint work with Paul Muhly.

S. Voss

Construction of Quantum L'evy Processes on Dual Groups

W. von Waldenfels

Law of Large Numbers and CLT for Anti-Commutative Tensor Independence

We consider a 2-graded algebra $\mathfrak{A}$ and odd elements $a_i \in \mathfrak{A}, i \in I$ and a 2-graded algebra $\mathfrak{B}$ and an *even* functional $\omega : \mathfrak{A} \to \mathfrak{B}$ and with $\omega(1) = 1$ and for a fixed s

$$\omega(a_{i_1}) = 0, \cdots, \omega(a_{i_1} \cdots a_{i_{s-1}}) = 0.$$

Denote by $\mathfrak{F}$ the free algebra $\mathbb{C}\langle X_i, i \in I\rangle$. We want to calculate for $f \in \mathfrak{F}$ and $N \to \infty$

$$F_N(f) = M_N \circ \omega^{\otimes N}(f(X_i \mapsto N^{-1/s}(a_i \otimes 1 \otimes \cdots \otimes 1 + 1 \otimes \cdots \otimes 1 \otimes a_i))$$

The limit exists and can be calculated by the means of an universal gaussian functional

$$\Gamma_s : \mathfrak{F} \to \mathfrak{L}_s$$

where for even s

$$\mathfrak{L}_s = \mathbb{C}\left[\Xi_{i_1,\cdots,i_s} : i_1, \cdots, i_s \in I\right]$$

is the commutative polynomial algebra generated by the $\Xi_{i_1,\cdots,i_s}$ and for odd s

$$\mathfrak{L}_s = \bigwedge \left(\Xi_{i_1,\cdots,i_s} : i_1, \cdots, i_s \in I\right)$$

the Grassmann algebra generated by the $\Xi_{i_1,\cdots,i_s}$.

Use instead of $\mathbb{C}$ the algebra $\mathfrak{L}_s$ as coefficient algebra of $\mathfrak{F}$ and denote by $\mathfrak{F}\mathfrak{L}^g$ the subspace of $\mathfrak{F}$ spanned by

$$X_i, [X_i, X_j]^g, [X_i, [X_j, X_k]^g]^g, [X_i, [X_j, [X_k, X_l]^g]^g]^g, \cdots .$$

Here $[f_1, f_2]^g$ is the 2-graded commutator. Define

$$\Theta_s : \mathfrak{F} \to \mathfrak{L}_s$$
$$\Theta_s(X_w) = \begin{cases} \Xi_w & \text{for } \#w = s \\ 0 & \text{for } \#w \neq s \end{cases}.$$

The functional Γ_s vanishes on the two-sided ideal $\mathfrak{I}$ generated by the elements of the form

$$f - \Theta_s(f)$$

where f runs through all homogeneous polynomials of degree s in $\mathfrak{F}\mathfrak{L}^g$. For $s = 1$ we have the law of large numbers and the quotient is a Grassmann algebra.That indicates, that the classical limits of odd quantities are Grassmann numbers and not observable. For $s = 2$ the limit is the central limit theorem and the quotient a Clifford algebra. So we get the famous anticommutation relations as result of the central limit theorem .

S.L. Wills

E-Semigroups Subordinate to CCR Flows

The subordinate E-semigroups of a fixed E-semigroup α are in one-to-one correspondence with local projection-valued cocycles of α . For the CCR flow we characterise these cocycles in terms of their stochastic generators, that is, in terms of the coefficient driving the quantum stochastic diff erential equation of Hudson-Parthasarathy type that such cocycles necessarily satisfy. In addition various equivalence relations and order-type relations on E-semigroups are considered, and shown to work especially well in the case of those semigroups subordinate to the CCR flows by exploiting our characterisation.

J. Wysoczanski

An Independence and Product of Algebras (and Hilbert Spaces) Indexed by Partially Ordered Sets